

An internally audited global-continuation certificate for one unforced periodic Navier-Stokes datum

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AI-assisted research: ChatGPT (OpenAI) and Claude (Anthropic)

PRELIMINARY RESEARCH RECORD | VERSION 1.0-preliminary

Prepared 14 October 2026

Internal, conditional research record. The finite certificate cannot be independently reproduced from this record alone.

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Abstract

We record an internally audited computer-assisted certificate for an explicit, smooth, unforced initial datum for the three-dimensional incompressible Navier–Stokes equations on the periodic torus. The recorded finite certificate covers the interval $[0, 23/40]$ and yields an upper bound 1.398126691309 for the Fourier–Wiener norm of the exact solution at its final time. The datum belongs to an invariant Fourier lattice with spectral gap $\sqrt{2}$. The strict final bound therefore implies global smooth continuation through a standard small-data argument, which is given below. The evidence consists of exact internal interval checks and a composed audit; its coverage and qualifications are stated explicitly. This preliminary record does not contain the complete raw coefficient chain or checker and is not a standalone reproducible proof. It makes no claim to solve the arbitrary-data Navier–Stokes regularity problem.

1. Equation, conventions and datum

Let $T^3 = (R/(2\pi\mathbb{Z}))^3$. We consider

$$\begin{aligned} \text{partial}_t u + (u \cdot \text{grad})u + \text{grad } p &= \Delta u, \\ \text{div } u &= 0, \\ u(x, 0) &= u_0(x), \end{aligned}$$

with viscosity $\nu=1$ and forcing $f=0$. All fields are real and periodic; the velocity has zero spatial mean. Fourier coefficients and norms are normalized by

$$\begin{aligned} \hat{u}(k) &= (2\pi)^{-3} \int_{[0, 2\pi]^3} u(x) \exp(-i k \cdot x) dx, \\ A_s(u) &= \sum_{[k \in \mathbb{Z}^3, k \neq 0]} |k|^s |\hat{u}(k)|, \end{aligned}$$

where the coefficient norm is the Euclidean norm on $\mathbb{C}^3$. The initial datum is

$$\begin{aligned} u_0(x, y, z) &= (-1, 1, 0) \cos(x+y) \\ &\quad + (0, 2, 0) \cos(2x) \\ &\quad + (0, -1, 1) \cos(y+z). \end{aligned}$$

Each coefficient is perpendicular to its wave vector, so the datum is divergence-free. Its Fourier support lies in

$$\Lambda = \{ k \in \mathbb{Z}^3 : k_1 + k_2 + k_3 \text{ is even} \}.$$

This additive lattice is preserved by Fourier convolution, the Laplacian and the Leray projection. Equivalently, the datum is invariant under the translation $x \rightarrow x + (\pi, \pi, \pi)$; local uniqueness preserves this symmetry of the exact smooth solution. Every nonzero k in Λ has $|k| \geq \sqrt{2}$. The rational number

$$\text{ell} = 1414213/1000000$$

is a strictly smaller positive lower bound for this gap: $\text{ell}^2 < 2$.

2. Recorded finite certificate

The internal certificate uses 23 consecutive intervals of length $1/40$. Its final time is $T=23/40$. The error estimate concerns the full equation: the unknown error is not truncated to the approximation's Fourier modes. The residual includes the full quadratic output support, including modes outside the approximation cutoff.

The recorded final bounds are

$$\begin{aligned} A_0(U(T)) &\leq 1284461/1000000, \\ A_0(u(T)-U(T)) &\leq 113665691309/1000000000000. \end{aligned}$$

Their sum gives

$$\begin{aligned} A_0(u(T)) &\leq \alpha = 1398126691309/1000000000000 \\ &= 1.398126691309. \end{aligned}$$

The strict margin is exact:

$$\ell - \alpha = 16086308691/1000000000000 > 0.$$

Here U denotes the certified approximation and u the exact solution to the original datum. A bound for U alone would not be sufficient. The recorded endpoint bound includes the accumulated error and the actual joins between successive polynomial approximations; the carried error is not reset.

The certificate's logical premise is stronger than accuracy at isolated sample times. It requires verified full residual identities, continuous norm enclosures and a comparison argument that extends the exact solution across every interval. These finite-certificate obligations are recorded as internally checked; this short disclosure does not reproduce all of their raw inputs or verification programs.

3. Certificate-to-global continuation lemma

Lemma. Suppose the exact smooth solution with the datum in Section 1 has been continued to $T=23/40$ by a valid full-equation certificate, with $A_0(u(T)) \leq \alpha$ as above. Assume the standard local smooth existence, uniqueness and continuation criterion based on a finite time integral of $\| \text{grad } u \|_\infty$ for periodic Navier–Stokes. Then this same solution continues smoothly for every $t \geq 0$. In particular, for $t \geq T$,

$$A_0(u(t)) \leq \alpha \exp(-\gamma(t-T)),$$

where

$$\begin{aligned} \delta &= 1 - \alpha/\ell > 0, \\ \gamma &= \delta \ell^2 > 0. \end{aligned}$$

Proof. While the solution is smooth, the usual Fourier–Wiener estimate gives

$$D^+ A_0 + A_2 \leq A_0 A_1.$$

Here D^+ denotes the upper right Dini derivative. On every compact time interval where u is smooth, uniform Fourier decay makes A_0 , A_1 and A_2 finite and continuous. The Fourier convolution formula, the contraction property of the Leray projection and the triangle inequality give

$$\begin{aligned} \sum_k |P_{\hat{}}[(u \cdot \text{grad})u](k)| \\ \leq \sum_{(p,q)} |q| |u_{\hat{}}(p)| |u_{\hat{}}(q)| = A_0 A_1. \end{aligned}$$

Here $P_{\hat{}}$ denotes the Fourier action of the Leray projection; all sums include the full Fourier support. Integrating the modewise norm inequalities over $[s,t]$, summing first over finite sets of modes and passing to the limit gives

$$\begin{aligned} A_0(u(t)) - A_0(u(s)) + \int_s^t A_2(u(r)) \, dr \\ \leq \int_s^t A_0(u(r)) A_1(u(r)) \, dr. \end{aligned}$$

The displayed Dini inequality follows from this integrated form by continuity, including at times when individual Fourier coefficients vanish. No improved one-half coefficient is used. On Lambda,

$$A_1 \leq A_2/\ell, \quad A_2 \geq \ell^2 A_0.$$

Writing $a=A_0(u)$ and $D=A_2(u)$, we obtain

$$D^+ a \leq -(1-a/\ell)D.$$

Choose beta with $\alpha < \beta < \ell$. As long as $a \leq \beta$, this inequality makes a nonincreasing. The standard first-exit argument therefore excludes a crossing of beta; equivalently, its integral form yields $a(t) \leq \alpha$ throughout the maximal smooth interval after T . Consequently

$$a(t) + \delta \int_T^t A_2(u(s)) \, ds \leq \alpha.$$

Using $A_2 \geq \ell^2 a$ gives the stated exponential decay. Also,

$$\int_T^t A_1(u(s)) \, ds \leq \alpha/(\delta \ell), \\ ||\text{grad } u(s)||_\infty \leq A_1(u(s)).$$

This bound is uniform up to any finite endpoint of the maximal smooth interval. The stated continuation criterion rules out a finite breakdown after T . Local uniqueness joins the continuation to the solution already covered by the finite certificate. No polynomial approximation is extrapolated beyond its certified interval. QED.

The lemma is a sufficient certificate-to-continuation implication. Its general Fourier estimate and small-data mechanism are not claimed as new. It is not a substitute for the finite-certificate premises in Section 2.

4. What the internal audit covers

The final single-interval stage, internally designated V33B, records a passing independent checker for its new final interval and a passing strict terminal test, conditional on the inherited prefix. A subsequent composed audit, internally designated V40, records the following broader coverage:

- Intervals 00–08: fresh independent checks of the full residual identities, structure, continuous enclosures, joins and scalar gates.
- Intervals 09–22: previously completed exact residual checks bound to the inspected raw-file hashes, with fresh checks of endpoints, joins, Bernstein norm enclosures, scalar gates and the accumulated error.
- The terminal bound and strict gap comparison stated in Section 2.

The recorded status is PASS_COMPOSED_INTERNAL_AUDIT. It is not a single fresh replay of every residual in the entire chain. One recovered interval uses an independently reconstructed mathematical payload; no claim is made that this payload reproduces the historical file bytes. The earlier V32 scalar gate retains its recorded status HOLD (not passed); the accepted continuation uses the separately stated V32B affine-barrier argument.

“Independent checker” here means a checker distinct from the generator in the internal computational workflow. It does not mean external peer review or verification by an unrelated institution. Reading and reconciling these records in October is also not a new execution of the archived certificate.

5. Evidence, chronology and scope

The verification evidence remains in the author's private research archive. This selected preliminary record omits the full raw coefficient chain and checker. A reader can inspect the continuation implication above, but cannot independently reproduce the finite certificate from this document alone.

Appendix A gives selected SHA-256 digests for the stored specification, results, handoff, raw coefficient files and V40 receipts. The digest of the intact slab-18 ZIP member is explicitly identified as recorded in the

preserved source notes, not freshly recomputed in preparing this revision. The other selected digests were computed from available bytes. These digests support future byte-for-byte comparison; they do not provide the omitted witness data or verifier, establish mathematical correctness, authenticate authorship, or certify the historical creation dates. The earlier truncated slab-18 copy is not the object identified in Appendix A.

The archived final-stage specification records the internal timestamp 2026-09-15T16:16:15.369287+00:00. The composed-audit report is internally dated 16 September 2026. These are descriptions of the preserved records, not independently trusted timestamps, proof of authorship or a claim of public disclosure on those dates. This draft is dated 14 October 2026.

The reported conclusion concerns one explicit initial datum. It neither covers arbitrary smooth data nor proves the general Navier–Stokes B statement. It is not a singularity construction. The internal certificate has not been externally peer-reviewed or verified in a formal proof assistant. Novelty of the specific application and its precise relationship to prior computer-assisted work remain to be assessed; the general Fourier–Wiener estimates and a posteriori methodology are not claimed as original discoveries here. In particular, a posteriori control of exact periodic Navier–Stokes solutions by approximate solutions, including sufficient global-existence tests, has a substantial prior literature; [1] is one relevant predecessor. This citation is not an exhaustive novelty search and does not certify the particular computation reported here.

Reference

[1] Carlo Morosi and Livio Pizzocchero, On approximate solutions of the incompressible Euler and Navier–Stokes equations, *Nonlinear Analysis* 75 (2012), 2209–2235. DOI: 10.1016/j.na.2011.10.020. <https://arxiv.org/abs/1104.3832> <https://doi.org/10.1016/j.na.2011.10.020>

AI-use disclosure

The author used ChatGPT (OpenAI) and Claude (Anthropic) for research assistance, including discussion and scrutiny of mathematical arguments and review of research records. ChatGPT assisted with reconciliation of the available sources, review of computational records, and preparation and revision of this report. For preparation of this revision, ChatGPT also prepared and used tool-executed scripts for archive inspection, byte-level hash comparisons and document formatting. These activities were evidence and document processing, not a fresh PDE simulation or residual replay.

In the documented exchange concerning this revision, Claude commented on publication strategy and reviewed the text and analytic continuation argument in Section 3. That review was not an independent verification or replay of the finite certificate in Section 2, and it was not external peer review. These descriptions identify documented contributions without assigning exclusive roles in earlier work. The available records do not establish an exhaustive tool-by-tool attribution for all original code generation and archived computations; no such exhaustive attribution is claimed. Computational conclusions remain limited to the execution and audit evidence described in this report.

Neither AI tool is listed as an author. The human author is responsible for the final report and its claims. This disclosure does not assert that the author personally reran every archived computation. Agreement between AI tools is not treated as mathematical certification.

Appendix A. Selected SHA-256 digests

These digests identify exact serialized file bytes. DIRECT means the bytes were hashed in this preparation. RECORDED means an expected digest from preserved V40 records; the raw bytes were not rehashed for this revision. This is a selected inventory, not a complete replay package.

S01. V33B_FROZEN_SPEC.json | 4,828 bytes | DIRECT

9b4aba0e728bccb2caae21e065426a986ae0d213c29f5b361702c51e7b8b3edf

Frozen specification record; byte check only.

S02. V33B_RESULT.json | 4,316 bytes | DIRECT

84819cbf14ebdc485666bd358dde792119281da31b051b3c9f5940caee2b09bf

Archive basename RESULT.json; selected-source alias V33B_RESULT_SLAB_22.json. Result receipt, not raw coefficients.

S03. V33B_GLOBAL_HANDOFF.json | 1,167 bytes | DIRECT

8a15aaf4161a926c2eb76538d9692cd6ff057f5db64d1c9864bfc9a87cecf03

Stored global-handoff record; byte check does not revalidate its premises.

S04. SLAB_00.json | 496,685 bytes | DIRECT

fef69374e095b7a73fd2a456f9b95460d6bf69246634f471743ed2ab4b682da7

Raw archive-member bytes match the V40 prefix receipt.

S05. SLAB_18.json (intact member; recorded digest) | 106,338,512 bytes | RECORDED

458915db66f49277ec9cab2aafbb74c639a92ff1071cf46169697be824dba5c5

Expected digest and byte length from preserved source notes. Raw member not located/rehashed here. Truncated copy excluded.

S06. SLAB_22.json | 11,223,242 bytes | DIRECT

5132eb4489e9df60fbe2515ee208e6f22d41aba990e88329026d2d99f30b4807

Raw archive-member bytes match the V40 composed receipt.

S07. V40_02_PREFIX_RESULT.json | 7,299 bytes | DIRECT

fb63c0dc66e97fd9888970b65bb7c91193142831a191b433ec8ecb1b5628197e

Receipt of historical V40 checks on 00-08; only receipt bytes checked here.

S08. V40_04_COMPOSED_CHAIN_RESULT.json | 8,348 bytes | DIRECT

3f2985128ccd85d0b111a3c9eea1a86927ad2cd6ae9c11e94126870e6ce1d28a

PASS_COMPOSED_INTERNAL_AUDIT receipt; historical coverage, not a new residual replay.

S09. V40_10_RESULT_MR.md | 9,355 bytes | DIRECT

d0c88e568c23384c1640c987ac079c1b5ba2ed4d2294c32ea824f67cb8536fa3

Preserved explanatory report; digest does not certify its mathematical assertions.

The slab-18 digest refers to the intact ZIP member described in the source records, not the truncated extracted copy. The raw member was not located in the targeted search for this revision; this does not establish global unavailability. Digests support future byte comparisons. They do not prove mathematical correctness, authorship, priority or trusted creation dates, and they do not replace omitted witness data and a verifier.